第四次作业

请于**十一月五日周二（校历第九周）**当堂上交本次作业。只收纸质版。晚交不收。可以与班上同学讨论合作，请参与合作的同学共同交一份作业，并署上所有参与者姓名。

1. Let $M := \{M_k\}_{k=0}^{\infty}$ be a logarithmically convex sequence of positive numbers; that is

$$M_k \leq M_m^{\frac{n-k}{n-m}} \cdot M_n^{\frac{k-m}{n-m}}, \quad \forall\, m < k < n.$$

Denote by $C^{\infty}(\mathbb{R})$ the set of infinitely differentiable functions on $\mathbb{R}$. Set

$$C_M(\mathbb{R}) = \left\{ u \,\middle|\, u \in C^{\infty}(\mathbb{R}), \quad \exists\, C_1, C_2 \text{ such that } \sup_{x\in\mathbb{R}} \left| \frac{d^k u}{dx^k}(x) \right| \leq C_1 C_2^k M_k, \ \forall k = 0, 1, 2, \cdots \right\}.$$

Suppose that

$$\sum_{k=1}^{\infty} \frac{1}{M_k^{1/k}} = \infty.$$

Prove that for any $u \in C_M(\mathbb{R})$ vanishing of infinite order at 0 must vanish identically on $\mathbb{R}$.