

第一次作业

请于九月二十四日周二 (校历第三周) 当堂上交本次作业。只收纸质版。晚交不收。可以与班上同学讨论合作, 请参与合作的同学共同交一份作业, 并署上所有参与者姓名。

1. Let f be a real-valued continuous function defined on the interval $[0, 1]$. Let $B_n(f)$ be the polynomial on $[0, 1]$ that assigns to x the value

$$\sum_{k=0}^n \binom{n}{k} x^k (1-x)^{n-k} f\left(\frac{k}{n}\right).$$

Then $B_n(f)$ tends uniformly to f as $n \rightarrow \infty$. If f is continuously differentiable on $[0, 1]$, then $B'_n(f)$ tends to f' uniformly as $n \rightarrow \infty$.