

第七次作业 (Circle Method)

请于四月十八日周五 (校历第九周) 当堂上交本次作业。请独立完成。如有参与讨论者, 请引用或致谢他们。

1. Denote by $p(n)$ the number of unrestricted partitions of n , that is,

$$1 + \sum_{n=0}^{\infty} p(n)x^n = \frac{1}{(1-x)(1-x^2)(1-x^3)\cdots}.$$

Prove that as $n \rightarrow \infty$, the asymptotic formula of $p(n)$ is

$$p(n) \sim \frac{1}{4n\sqrt{3}} \exp\left(\pi\sqrt{\frac{2n}{3}}\right).$$